Set II Std : 12^{th} HSC	**Subject – Maths**	Time: 3 Hrs Marks : 80

Section A

Q.1 Select and write the correct answer **16**

(i) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} =$

a) $2\sqrt{x} - \sqrt[3]{x} + \sqrt[6]{x} - log\left|\sqrt[6]{x}+1\right| + c$

b) $2\sqrt{x} - 3\sqrt[3]{x} + 2\sqrt[6]{x} - 6log\left|\sqrt[6]{x}+1\right| + c$

c) $2\sqrt{x} + 3\sqrt[3]{x} + \sqrt[6]{x} - 6log\left|\sqrt[6]{x}+1\right| + c$

d) $2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6log\left|\sqrt[6]{x}+1\right| + c$

(ii) If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $n \in N$, $i = \sqrt{-1}$ then $A^{4n} =$

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ c) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(iii) The equation of tangent to the curve $y^2 = 16x$ at the point $(4, -8)$ is

a) $x + y - 4 = 0$ b) $x - y + 4 = 0$

c) $x + y + 4 = 0$ d) $x - y - 4 = 0$

(iv) If the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represents a pair of lines, then the value of $'k' =$

a) $\frac{35}{4}, 5$ b) $-\frac{35}{4}, -5$ c) $\frac{35}{4}, -5$ d) $-\frac{35}{4}, 5$

(v) If $y = \tan^{-1}\left(\frac{2\sqrt{x}}{1+3x}\right), \frac{dy}{dx} =$

a) $\frac{1}{\sqrt{x}}\left(\frac{1}{1+9x} - \frac{1}{1+x}\right)$ b) $\frac{1}{\sqrt{x}}\left(\frac{3}{1+9x} - \frac{1}{1+x}\right)$

c) $\frac{1}{2\sqrt{x}}\left(\frac{1}{1+9x} - \frac{1}{1+x}\right)$ d) $\frac{1}{2\sqrt{x}}\left(\frac{3}{1+9x} - \frac{1}{1+x}\right)$

(vi) The value of 'p' if the vectors $\bar{\imath} - 2\bar{\jmath} + \bar{k}, 2\bar{\imath} - 5\bar{\jmath} + p\bar{k}$ *and* $5\bar{\imath} - 9\bar{\jmath} + 4\bar{k}$ are coplanar is

a) 1 b) 2 c) 3 d) 4

(vii) $\sim(p \to \sim q) \equiv$

a) $\sim p \to q$ b) $p \vee q$ c) $p \wedge q$ d) $\sim(p \to q)$

(viii) The approximate value of $\sqrt[3]{66}$ is

a) 4.01 b) 4.2 c) 4.3 d) 4.04

Q.2 Answer the following **4**

(i) Find the order and degree of the differential equation: $\frac{d^3y}{dx^3} = \left[1 + \left(\frac{d^2y}{dx^2}\right)^3\right]^2$

(ii) If in a triangle ABC, $a = 12, b = 5, c = 13$, show that it is a right-angled triangle.

(iii) If the mean and variance of a binomial distribution are 18 and 12, find n.

(iv) Write in symbolic form: If I drive fast and do not follow traffic rules, then I will surely meet with an accident.

Section B

Attempt any Eight **16**

Q.3 Find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ (using definite integration)

Q.4 If $x = 3sin\theta - 2sin^3\theta, y = 3cos\theta - 2cos^3\theta$, then show that $\frac{dy}{dx} = 1$, *when* $\theta = \frac{\pi}{4}$.

Q.5 Find the vector equation of a plane given by $2x + 3y - z = 7$

Q.6 Find the vector and Cartesian equation of the line Passing through the point (1,2,3) and perpendicular to the vectors $\bar{i} + \bar{j} + \bar{k}$ and $2\bar{i} - \bar{j} + \bar{k}$

Q.7 Show that the points A(1,-2,3),B(2,3,-4) and C(0,-7,10) are collinear.

Q.8 Find the principal solution for $cosx = -\frac{\sqrt{3}}{2}$

Q.9 If $xe^{2y} + ye^{2x} = 1$,find $\frac{dy}{dx}$.

Q.10 Find the centroid of the tetrahedron with vertices $A(3,-5,1), B(5,4,2), C(7,-7,3), D(1,0,2)$.

Q.11 Evaluate: $\int \frac{x+sinx}{1-cosx} dx$

Q.12 Evaluate: $\int_0^1 \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$

Q.13 State which of the following sentences are statements. In case of statements write their truth values.

(a)12 is a prime number.

(b)$x^2 - 3x - 4 = 0$

Q.14 The p.d.f. of a continuous random variable X is

$f(x) = \frac{x}{8} \qquad 0 < x < 4$

$\quad = 0 \qquad otherwise.$

Find $P(X \leq 2), P(X > 3)$

Section C

Attempt any Eight **24**

Q.15 Solve the differential equation: $cos^2(x^2 + y^2) = x + y\frac{dy}{dx}$,when x = 0, y = 0

Q.16 Solve: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

Q.17 Find the co-ordinates of the point which divides the line segment joining the points A(3,4,3) and B(2,2,7) in the ratio 2:5 externally.

Q.18 Using vector methods show that the quadrilateral with vertices P (1,2,-1), Q (8, -3, -4), R (5, -4,1) and S (-2,1,4) is a parallelogram.

Q.19 Find the direction cosines of the line $3x - 1 = 6y + 2 = 2z - 3$.Hence find the equation of a line passing through the point (2,1,3) and parallel to the above line.

Q.20 A random variable X has the following probability distribution.

x	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

Find (a)k (b) $P(X \geq 2)$ (c)c.d.f. of X.

Q.21 If $y = \left(x + \sqrt{x^2 - 1}\right)^m$ then show that

$(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m^2y$

Q.22 A box of square base and open top is to be made whose surface area is 192sq.cm.Find its dimensions so as to have maximum volume.

Q.23 Find: $\int \frac{1}{4+5sinx} dx$

Q.24 If u and v are two functions of x then prove that

$\int uv\, dx = u \int v\, dx - \int \left[\frac{du}{dx} \int vdx\right] dx$. Hence find $\int xsinxdx$

Q.25 Construct a truth table: $[(p \vee q) \vee (p \wedge r)] \rightarrow \sim p$

Q.26 Find the co-ordinates of the foot of the perpendicular from the origin to the plane $2x - y - 2z = 27$.

Section D

Attempt any five. **20**

Q.27 If $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 6 & 2 \\ 7 & 0 & -1 \\ -7 & -3 & 2 \end{bmatrix}$, find a matrix X such that $XA = B$.

Q.28 Find the joint equation of a pair of lines through the origin and perpendicular to the pair of lines $3x^2 + 4xy - 3y^2 = 0$

Q.29 In the old days, there was a probability of 0.8 of success in any attempt to make a telephone call. Calculate the probability of having 7 successes in 10 attempts.

Q.30 An ice ball melts at the rate which is proportional to the amount of ice at that instant. If half the quantity melts in 20 minutes, show that after 1 hour the amount of ice left will be $\left(\frac{1}{8}\right)^{th}$ of the original.

Q.31 Find the values of x for which the function $f(x) = 2x + \frac{1}{2x}$ is
(a) increasing (b) decreasing

Q.32 Prove : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Q.33 Maximize: z = 7x+11y subject to $3x + 5y \le 26, 5x + 3y \le 30,$
$x \ge 0, y \ge 0.$

Q.34 In $\Delta ABC, If\ m\angle C = 90^\circ\ then\ prove : tan\frac{B}{2} = \frac{1 - tan\frac{A}{2}}{1 + tan\frac{A}{2}}$